OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS — 4721

Core Mathematics 1

Monday **16 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
- 8 page answer booklet
- Graph paper
- List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

WARNING

You are not allowed to use a calculator in this paper.

This question paper consists of 3 printed pages and 1 blank page.

1 Solve the equations

(i) $x^{\frac{1}{3}} = 2$, [1]

(ii) $10^t = 1$, [1]

(iii) $\left(y^{-2}\right)^2 = \frac{1}{81}$. [2]

2 (i) Simplify $(3x+1)^2 - 2(2x-3)^2$. [3]

(ii) Find the coefficient of x^3 in the expansion of

$$(2x^3 - 3x^2 + 4x - 3)(x^2 - 2x + 1).$$ [2]

3 Given that $y = 3x^5 - \sqrt{x} + 15$, find

(i) $\frac{dy}{dx}$, [3]

(ii) $\frac{d^2y}{dx^2}$. [2]

4 (i) Sketch the curve $y = \frac{1}{x^2}$. [2]

(ii) Hence sketch the curve $y = \frac{1}{(x-3)^2}$. [2]

(iii) Describe fully a transformation that transforms the curve $y = \frac{1}{x^2}$ to the curve $y = \frac{2}{x^2}$. [3]

5 (i) Express $x^2 + 3x$ in the form $(x+a)^2 + b$. [2]

(ii) Express $y^2 - 4y - \frac{11}{4}$ in the form $(y+p)^2 + q$. [2]

A circle has equation $x^2 + y^2 + 3x - 4y - \frac{11}{4} = 0$.

(iii) Write down the coordinates of the centre of the circle. [1]

(iv) Find the radius of the circle. [2]

6 (i) Find the coordinates of the stationary points on the curve $y = x^3 - 3x^2 + 4$. [6]

(ii) Determine whether each stationary point is a maximum point or a minimum point. [3]

(iii) For what values of x does $x^3 - 3x^2 + 4$ increase as x increases? [2]

7 (i) Solve the equation $x^2 - 8x + 11 = 0$, giving your answers in simplified surd form. [4]

(ii) Hence sketch the curve $y = x^2 - 8x + 11$, labelling the points where the curve crosses the axes. [3]

(iii) Solve the equation $y - 8y^{\frac{1}{2}} + 11 = 0$, giving your answers in the form $p \pm q\sqrt{5}$. [4]

8 (i) Given that $y = x^2 - 5x + 15$ and $5x - y = 10$, show that $x^2 - 10x + 25 = 0$. [2]

(ii) Find the discriminant of $x^2 - 10x + 25$. [1]

(iii) What can you deduce from the answer to part (ii) about the line $5x - y = 10$ and the curve $y = x^2 - 5x + 15$? [1]

(iv) Solve the simultaneous equations

$$y = x^2 - 5x + 15 \quad \text{and} \quad 5x - y = 10.$$ [3]

(v) Hence, or otherwise, find the equation of the normal to the curve $y = x^2 - 5x + 15$ at the point (5, 15), giving your answer in the form $ax + by = c$, where a, b and c are integers. [4]

9 The points A, B and C have coordinates (5, 1), (p, 7) and (8, 2) respectively.

(i) Given that the distance between points A and B is twice the distance between points A and C, calculate the possible values of p. [7]

(ii) Given also that the line passing through A and B has equation $y = 3x - 14$, find the coordinates of the mid-point of AB. [4]